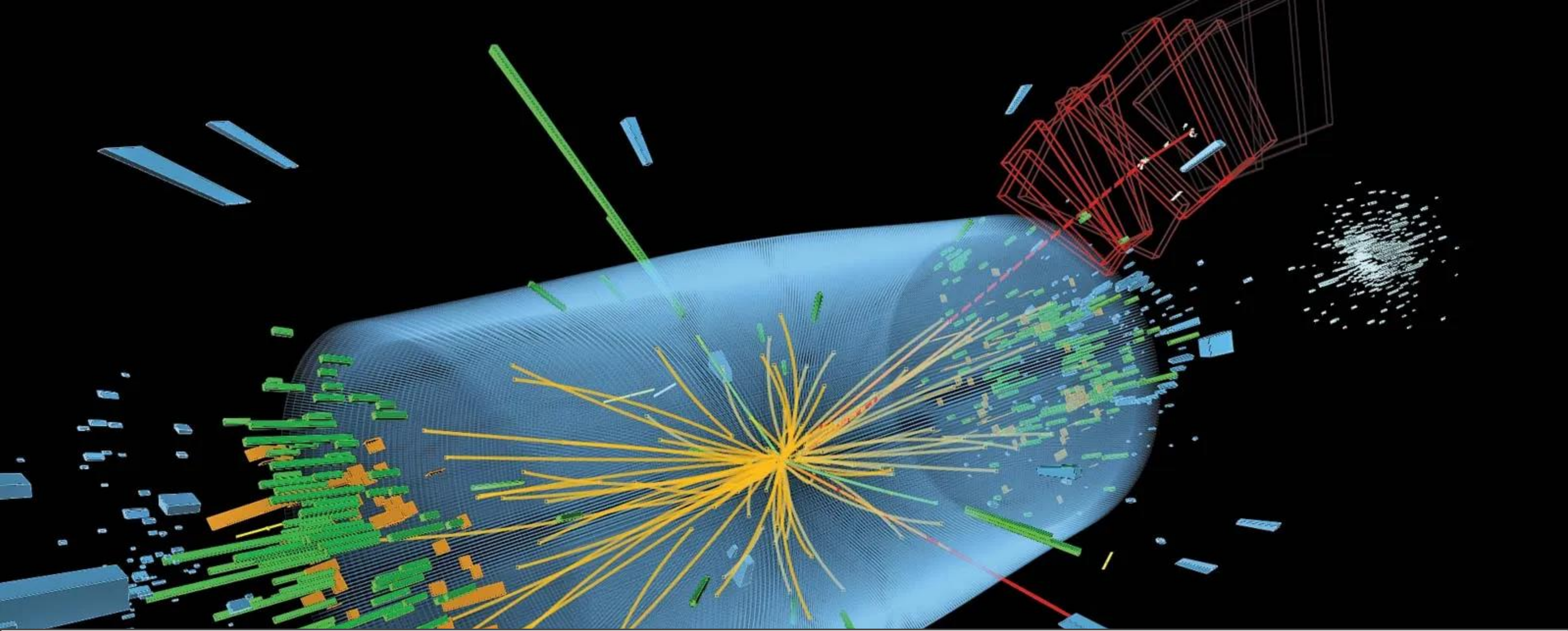


Particle Physics I

Lecture 2: Rutherford Scattering, Particle decays and **non**elementary particles

Prof. Radoslav Marchevski

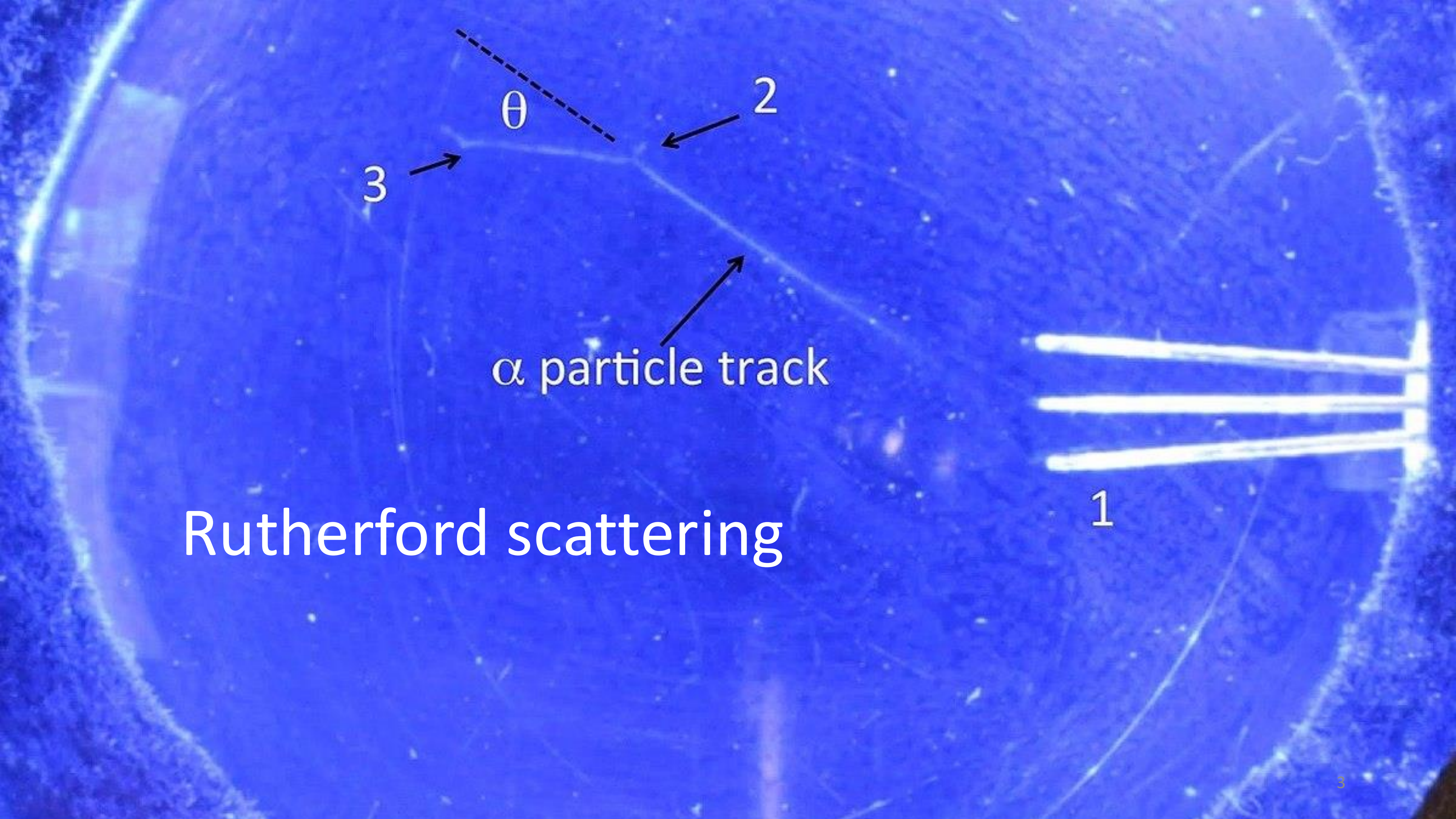
September 17th 2025

Today's learning targets

- Rutherford scattering as an example of how to do research in particle physics
  - What are the methods of particle physics?

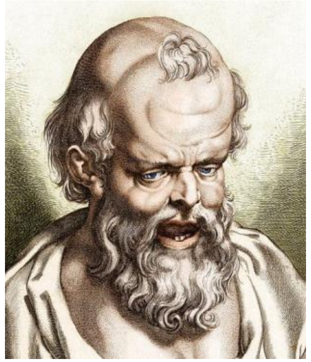
- Particles decays, examples of weak strong and electromagnetic decays
- What kind of nonelementary particles exist how are they classified
- How do we determine the physics properties of particles experimentally?

Rutherford scattering

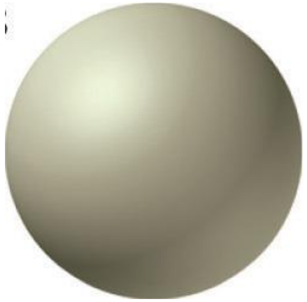


History of the atomic model

Democritus
460 BC



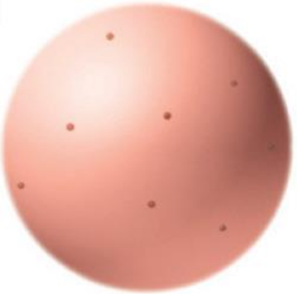
Dalton
1803 AD



Billiard ball model

The atom as a solid sphere with no positive or negative charge

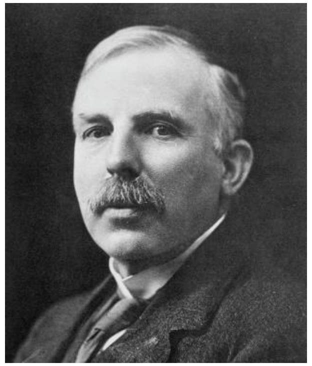
Thomson
1897

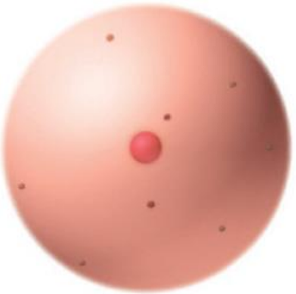


Plum pudding model

The sphere is positive, the red dots are negative charges embedded in it

Rutherford
1912

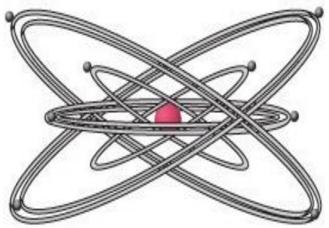




Rutherford model

Positive center + negative charges around it and empty space in between

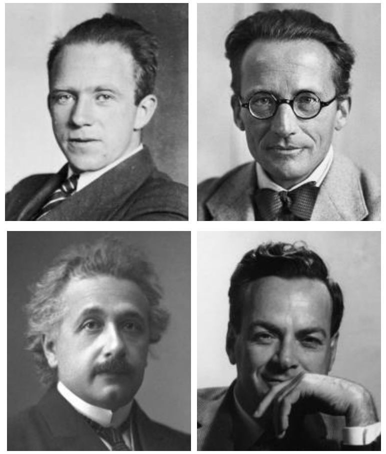
Bohr
1913

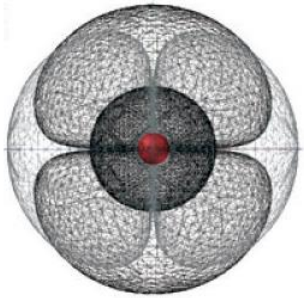


Bohr model

Electrons travel on definite trajectories around the nucleus and can jump from level to level

Heisenberg
Schroedinger
post-1930





Quantum cloud model

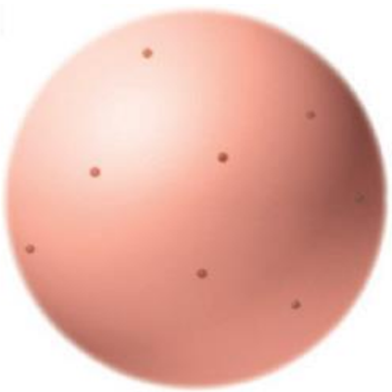
Electrons do not travel on definite trajectories. They are likely to be in regions (clouds)

https://www.atomictimeline.net

1904: The Thomson plum pudding model of the atom

- The discovery of the electron initiated models of the internal structure of the atom
- Atoms are neutral, therefore the rest of the atom must be positively charged

Thomson proposes a **"plum pudding model":**
negative electrons reside within positively charged substance

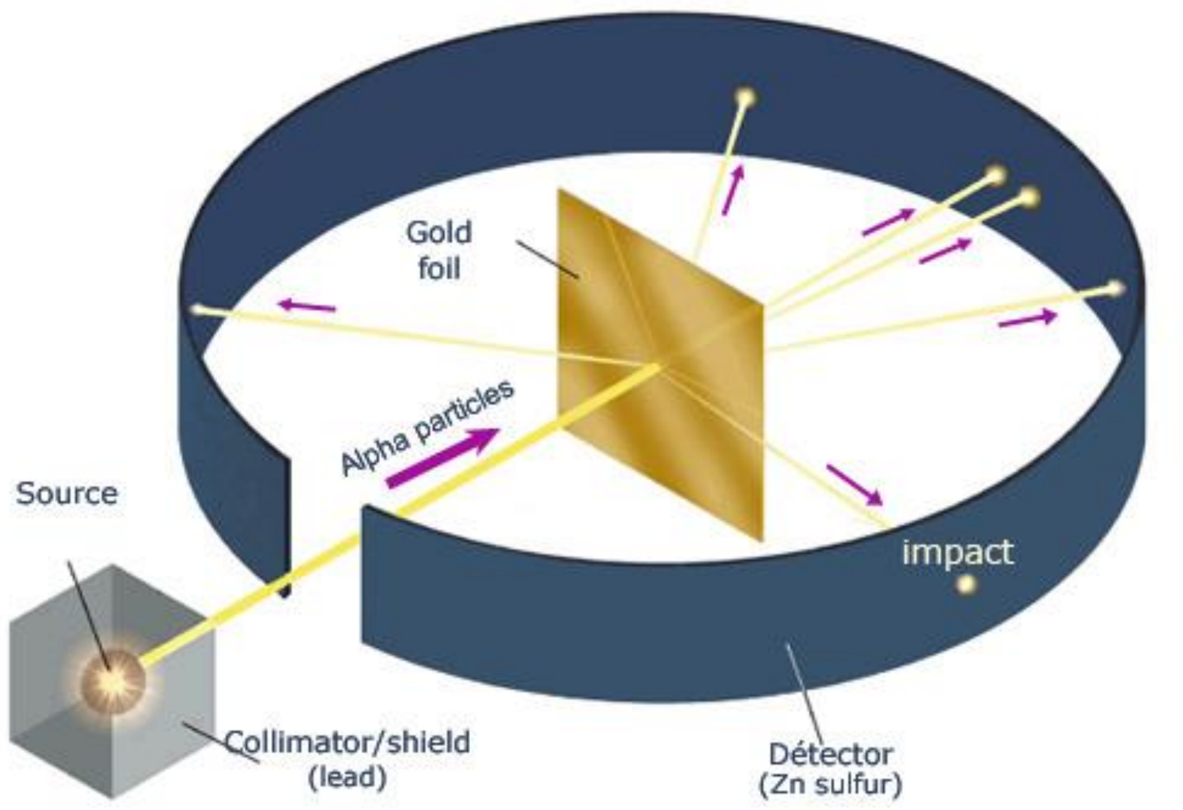



Hence matter should be transparent to incoming particles

- P. Lenard saw that materials were transparent to cathode rays (electrons)
- E. Rutherford observed the same with α −particle beam (He^{+2})

Scattering experiments: 1908-1913

E. Rutherford, H. Geiger, and E. Marsden



- Rutherford and his collaborators set up an experiment to investigate the structure of the atom by scattering α −particles on a thin gold foil
- They observed that a tiny fraction of the α −particles were scattered backwards by more than 90° degrees, while others suffered hardly any deflection
- The finding supports a model of mostly empty atoms with a small core where all the positive charges is concentrated (the nucleus)

We will compute basic properties of scattered particles in this experiment, and obtain the *differential cross section* of the process

Cross section definition

$$\sigma = \frac{\text{number of interactions per unit time per target}}{\text{incident flux}}$$

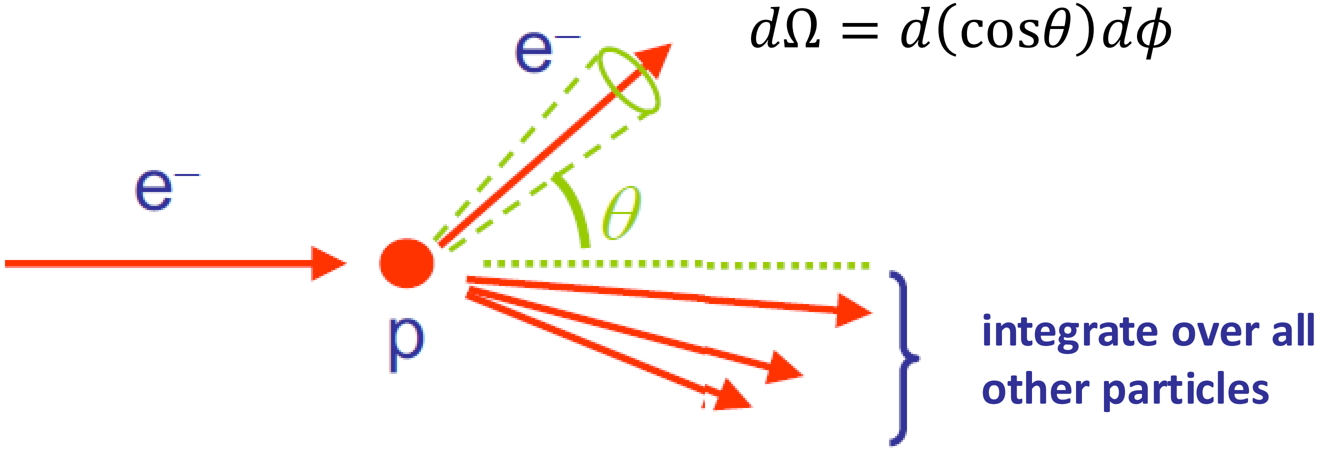
Incident flux = number of incident particles/unit area/unit time

- The "cross section", σ, can be thought of as the effective cross-sectional area representing the size of the target object that the incoming particles must hit for the interaction to occur
- It is a measure of the probability of the interaction
- In general, this has nothing to do with the physical size of the target although there are exceptions, e.g. neutron absorption

Classical analogue: for two bodies (e.g. balls), area transverse to their relative motion within which they scatter (i.e. hit each other)

Differential cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\text{number of interactions per unit time per target into a solid angle d}\Omega}{\text{incident flux}}$$



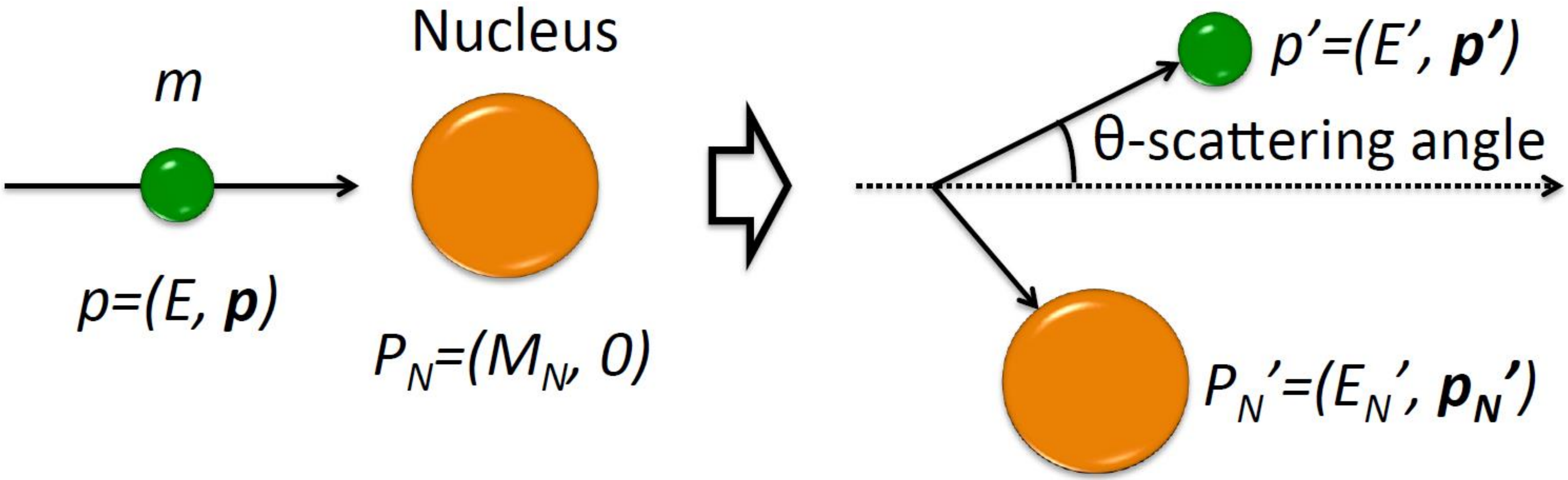
$$d\Omega = d(\cos\theta)d\phi$$

$$\sigma = \int \frac{\mathrm{d}\sigma}{d\Omega} d\Omega$$

Relativistic kinematics (reminder – more in exercise session)

- Space-time location and momentum-energy described by 4-component vectors
  - $X = (x_0, x_1, x_2, x_3, x_4) = (ct, \vec{x}) = (t, \boldsymbol{x})$
  - $P = (p_0, p_1, p_2, p_3, p_4) = (E/c, \vec{p}) = (E, \boldsymbol{p})$
- Scalar product of two four vector is Lorentz-invariant, i.e. independent of the reference frame
  - $a.b = ab = a_0 b_0 - a_1 b_1 - a_2 b_2 - a_3 b_3 = a_0 b_0 - \boldsymbol{ab}$
- For a 4-momentum it leads to
  - $P^2 = E^2 - \boldsymbol{p}^2 = m^2,$ m is a constant, corresponding to the mass of a particle at rest
- Energy-momentum conservation
  - $\sum_i P_i = const \implies m^2 = (\sum_i P_i)^2 = const$

Kinematics of an elastic scattering (I)

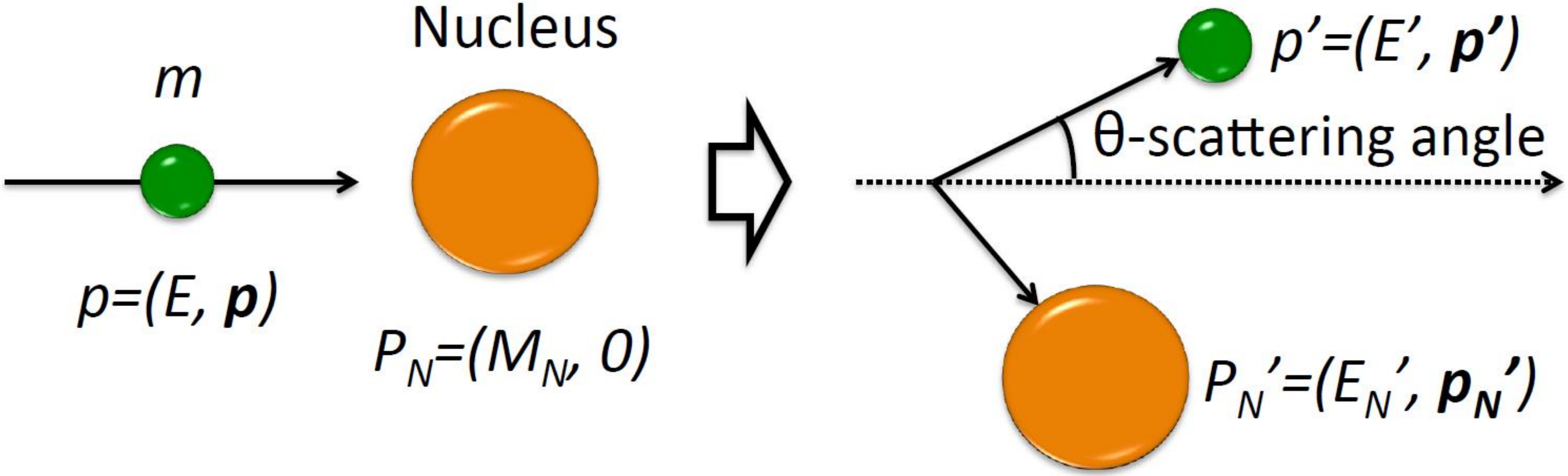


- From energy-momentum conservation

$$(p + P_N)^2 = (p' + P_N')^2$$
$$p^2 + 2pP_N + P_N^2 = {p'}^2 + 2p'P_N' + P_N'^2$$
$$p^2 = p'^2 = m^2 \text{ and } P_N^2 = P_N'^2 = M_N^2$$

$$\Longrightarrow \quad pP_N = p'P_N'$$

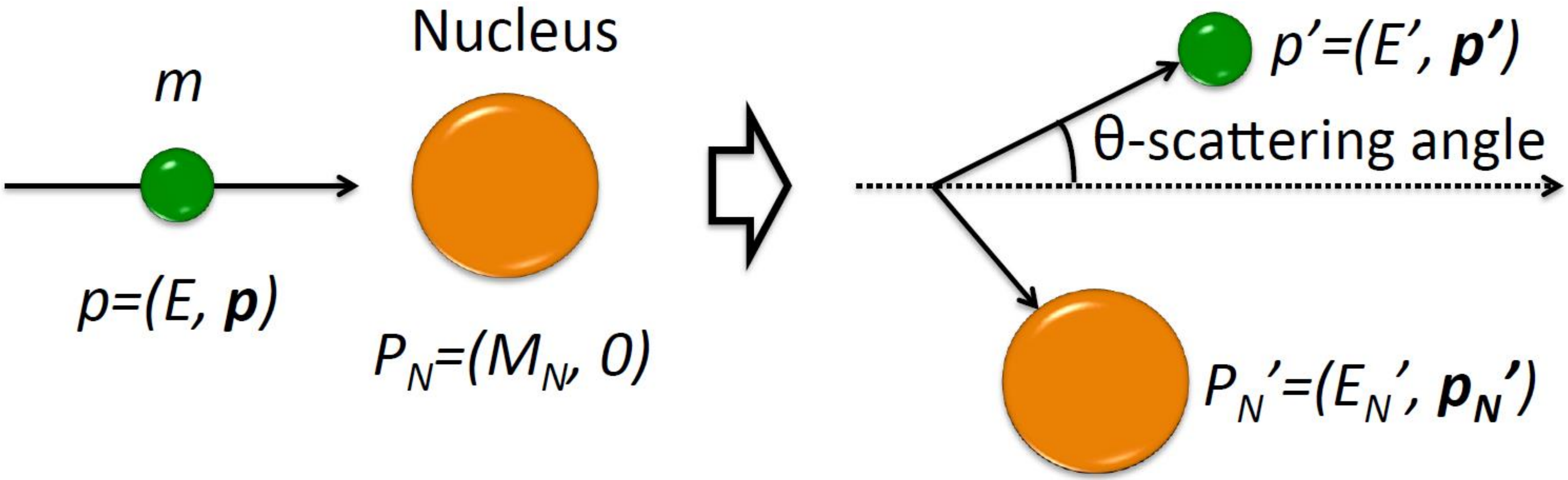
Kinematics of an elastic scattering (II)



- For a scattered particle with momentum p'

$$pP_N = p'(p + P_N - p') = p'p + p'P_N - m^2$$
$$EM_N = (E'E - \boldsymbol{p}'\boldsymbol{p}) + E'M_N - m^2$$

Kinematics of an elastic scattering (III)



- If $m \ll E$ then $|\boldsymbol{p}| \approx E$ and $|\boldsymbol{p}'| \approx E'$ so m can be neglected $\Longrightarrow$ $EM_N = EE'(1 - \cos\theta) + E'M_N$
- For a scattered particle in the laboratory frame: $$\boldsymbol{E' = \frac{E}{1 + \frac{E}{M_N}(1 - \cos\theta)}}$$

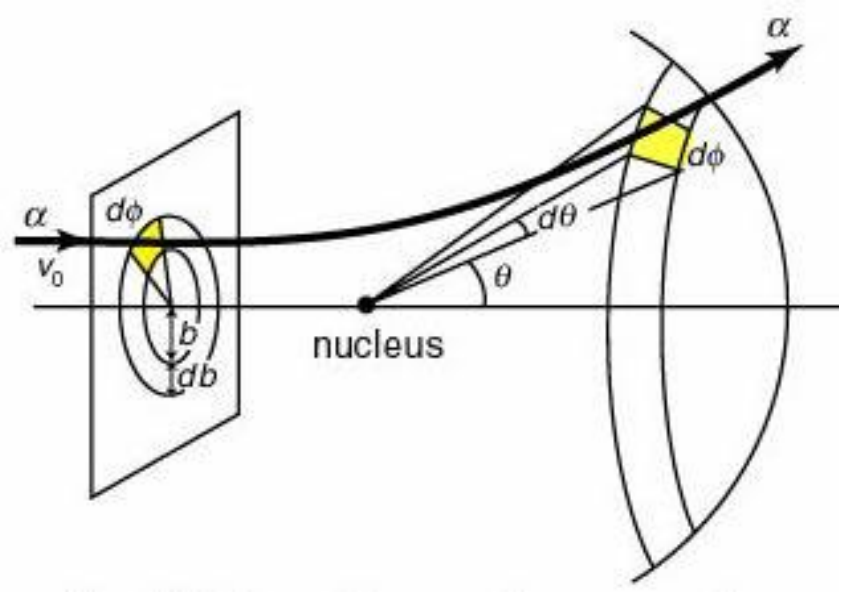
- M_N is large: small energy transfer $E - E'$ to the nucleus for all angles
- M_N is small: large energy transfer at large θ

Rutherford cross section

- Experimentally we measure the probability σ of a particle to scatter at a solid angle $d\Omega = \sin\theta d\theta d\phi$

$$dN = N\sigma(\theta)\,\sin\theta\; d\theta d\phi$$

- We can derive the following expression for the cross section from classical mechanics



$$\boldsymbol{\sigma(\theta) = \left(\frac{zZ\alpha}{4E_{\mathrm{kin}}}\right)^2 \frac{1}{sin^4(\theta/2)}}$$

Will be derived later in the course

- Experimentally we measure the probability $\sigma(\theta)$ is the differential Rutherford cross section for the scattering of a particle with charge ze and kinetic energy E_{kin} on a target nucleus with charge Ze

Relativistic Rutherford cross section

- The cross section holds in the relativistic case in the limit of negligible recoil energy from the target nucleus
- Rutherford cross section in the more general case

$$\sigma(\theta) = (zZ\alpha)^2 \frac{E'^2}{|\boldsymbol{q}|^4}, \text{where } \alpha = e^2, \boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}$$

- $\frac{1}{|\boldsymbol{q}|^4} \Longrightarrow$ very low event rates for particle scattering with large momentum transfer

Full Rutherford cross section

- Let's compute the full Rutherford cross section for $\theta > \theta_0$

$$\boldsymbol{\sigma}_{\mathbf{full}} = \int_{\theta_0}^{\pi} \sigma(\theta)\, 2\pi\, \sin\theta d\theta$$

$$= \int_{\theta_0}^{\pi} \sigma(\theta)\, 4\pi \frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)}\, d \sin\left(\frac{\theta}{2}\right) \text{ (substituting } \theta \text{ with } \sin\left(\frac{\theta}{2}\right)\text{)}$$

$$= \int_{\theta_0}^{\pi} \left(\frac{zZ\alpha}{4E_{\mathrm{kin}}}\right)^2 \frac{8\pi \sin\left(\frac{\theta}{2}\right) d \sin\left(\frac{\theta}{2}\right)}{\sin^4\left(\frac{\theta}{2}\right)} \text{ (using } \sin\theta = 2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right)\text{)}$$

$$= \left(\frac{zZ\alpha}{4E_{\mathrm{kin}}}\right)^2 \frac{-4\pi}{\sin^2\left(\frac{\theta}{2}\right)}\Bigg|_{\theta_0}^{\pi} \text{ (using } 1 - \frac{1}{\sin^2\left(\frac{\theta}{2}\right)} = \cot^2\left(\frac{\theta}{2}\right)\text{)}$$

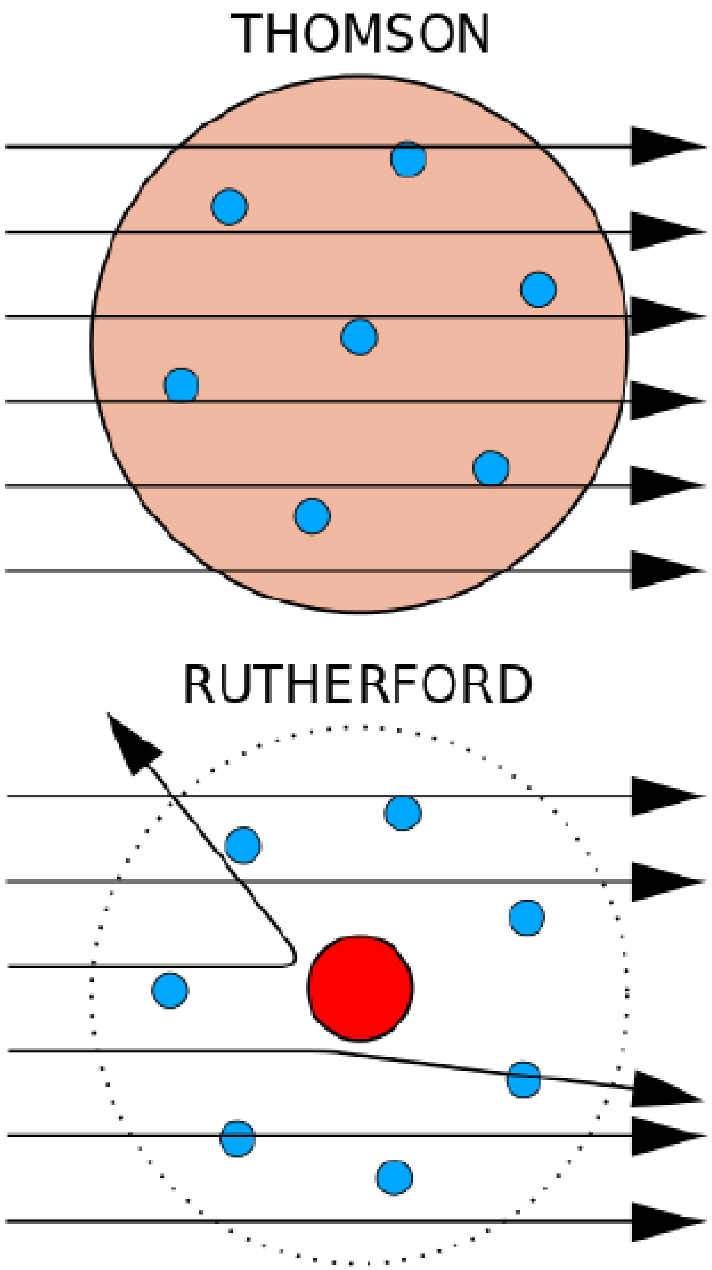
$$= \boldsymbol{4\pi \left(\frac{zZ\alpha}{4E_{kin}}\right)^2 \cot^2\left(\frac{\theta}{2}\right)}$$

Full Rutherford cross section

$$\boldsymbol{\sigma_{\text{full}} = 4\pi\left(\frac{zZ\alpha}{4E_{kin}}\right)^2 \cot^2\left(\frac{\theta}{2}\right)}$$

- For $\theta \to 0$ the cross section diverges: $\sigma_{\text{full}} \to \infty$
- Physics considerations: the larger the ***b*** value (impact parameter), the smaller the $\theta \Longrightarrow$ **all** distant particles remain undeflected ($\theta = 0$)
- Consequently, the formula can not be used for very small angles
  - $\theta = 0 \equiv$ nothing happens, we don`t include it in the full cross section
  - In practice, for large impact parameter ***b*** we would also need to take into account the screening due to the Coulomb potential of the electrons, and the presence of other nuclei in the target

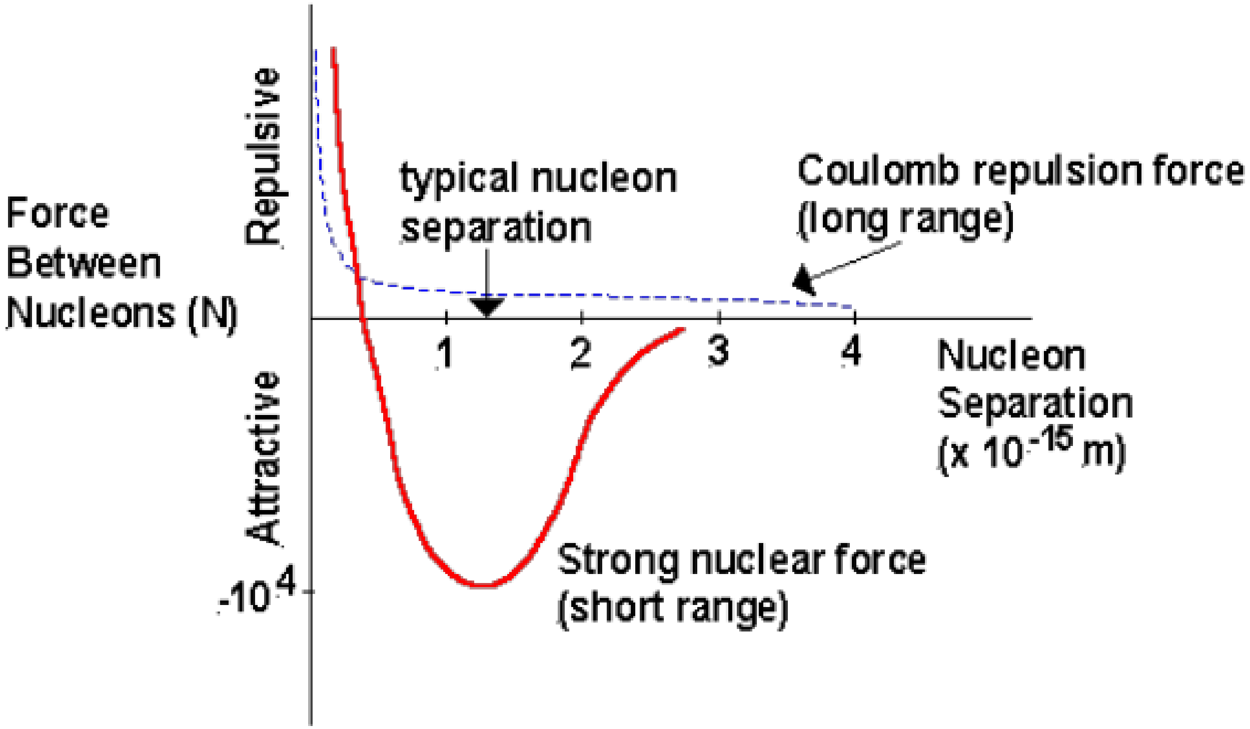
Backscattering from the nucleus



- Observation of α −particles scattered back from the gold foil led to the conclusion that there **must be a heavy point-like nucleus inside the atom**

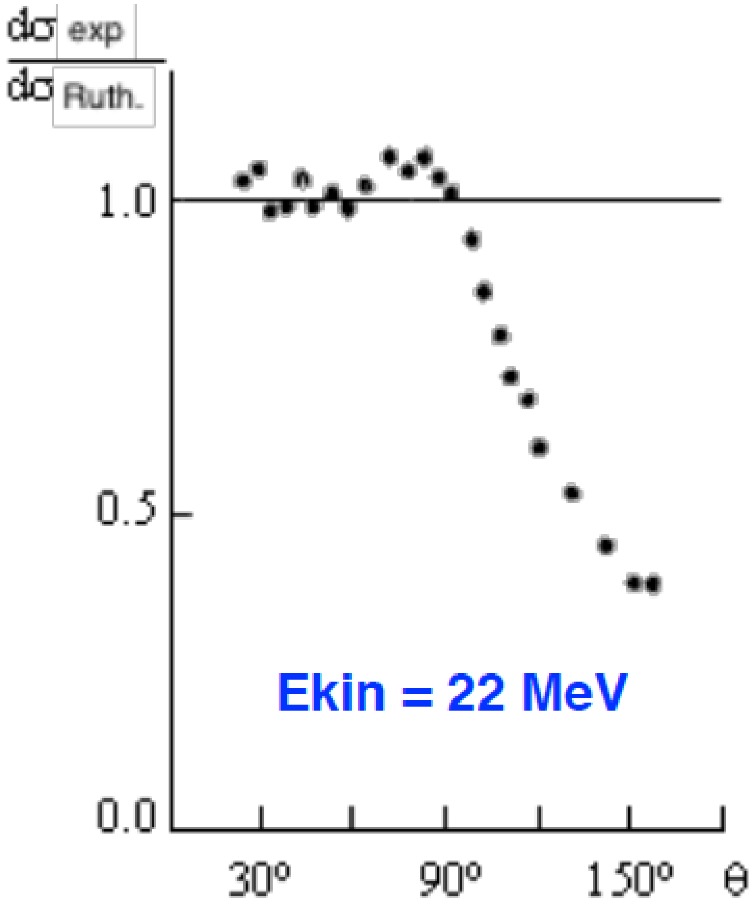
Coulomb law test

- At the same time as the Rutherford experiment, the Coulomb law was not yet tested at such small distances
- The successful description of the experimental results is an **indirect test of Coulomb's law** at the length scale of the nucleus ($\sim 10^{-15}\,\mathrm{m}$)
- Future tests at accelerators showed the limitations of such a test because the strong force becomes important at high energies (probed distance $\lambda \propto E^{-1}$)



Nucleus size determination

- α −particle with energy of ~ 7.7 MeV (Rutherford formula still works)
- Closest distance of approach greater than or equal to the size of the nucleus
- The kinetic energy of the system at the moment of "collision" is negligible $\approx p^2/2(M+m)$
- $2Ze^2/R = E_{\text{kin}}, Z_{AU} = 79$
  - remember $e^2 = \alpha = \frac{1}{137}$ and $\hbar c = 200$ MeV fm
- $R_{AU} + r_\alpha \leq R = \frac{2Z\alpha}{E_{\text{kin}}} = \frac{2\times\frac{79}{137}}{7.7\ \text{MeV}} \times 200\ \text{MeV}\cdot\text{fm} = 30\ \text{fm} = 3.0\times 10^{-14}\text{m}$
- Experimentally: Already for 22 MeV α −particles nuclear forces play a significant role
- **$\Longrightarrow 10^{-14}$ m is a typical size of a nucleus**

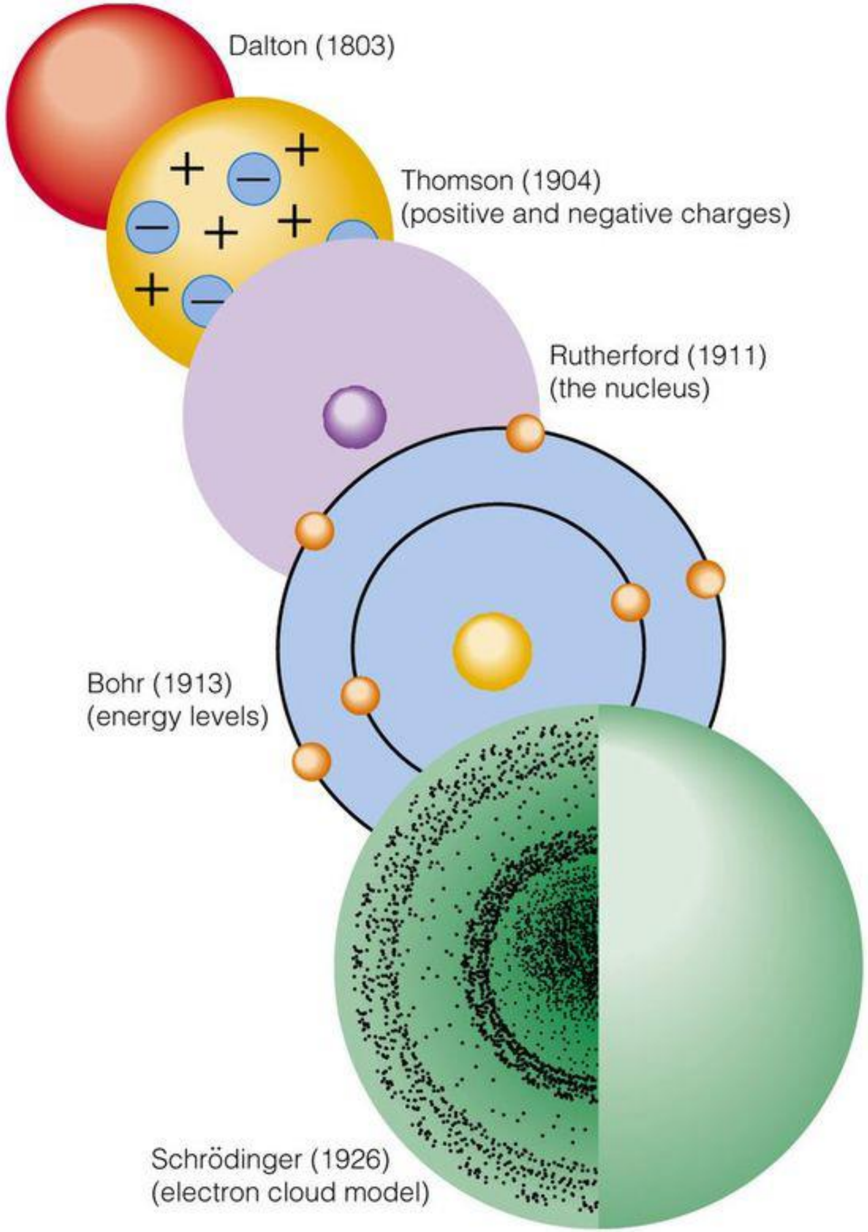


$R \approx R_0 A^{1/3}$
$R_0 \approx 1.2$ fm

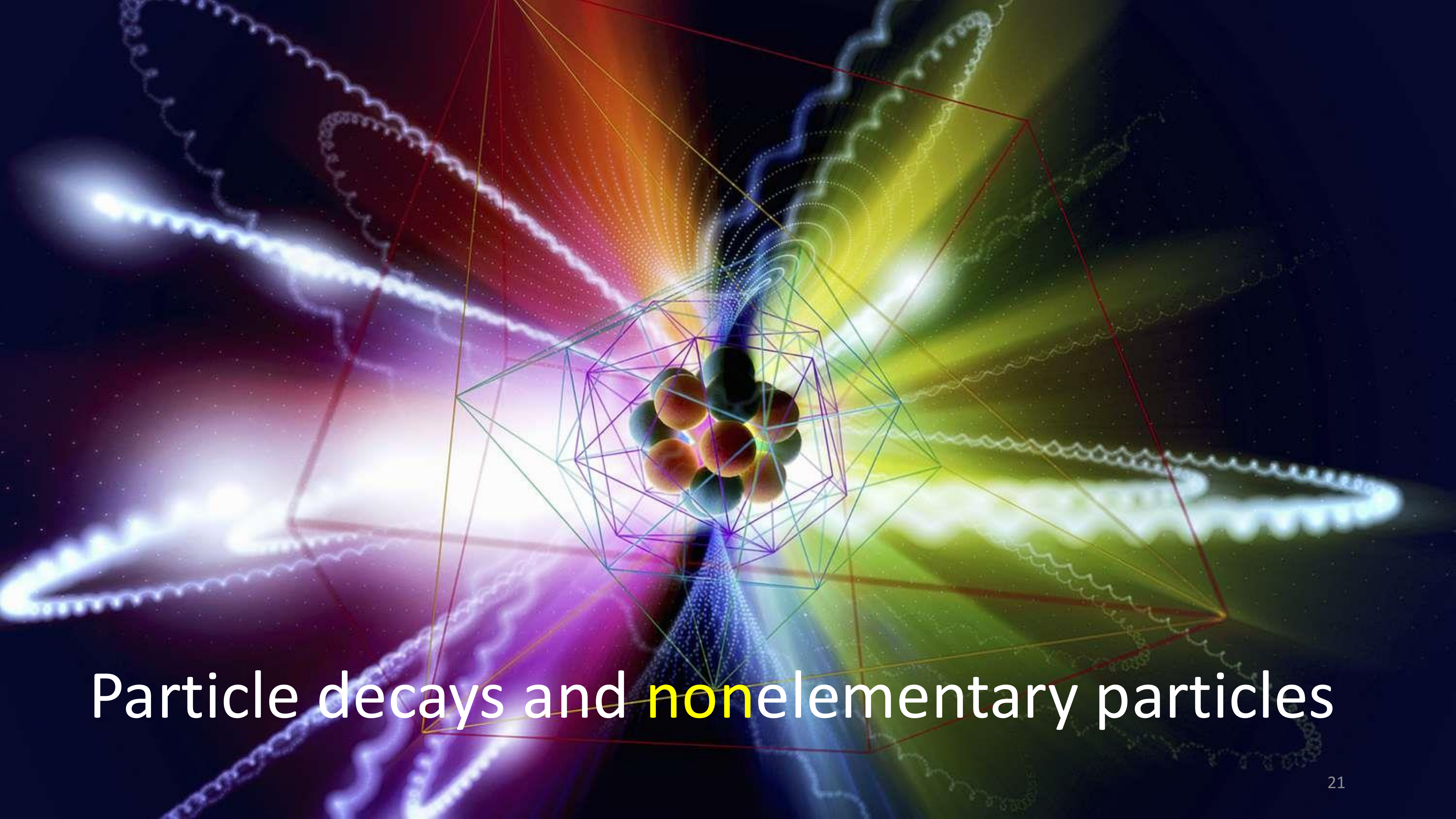
True AU nucleus radius ~ 7.3 fm

The 7.7 MeV α particles can`t penetrate the Coulomb potential and start probing the nuclear size

Summary of Rutherford experiment

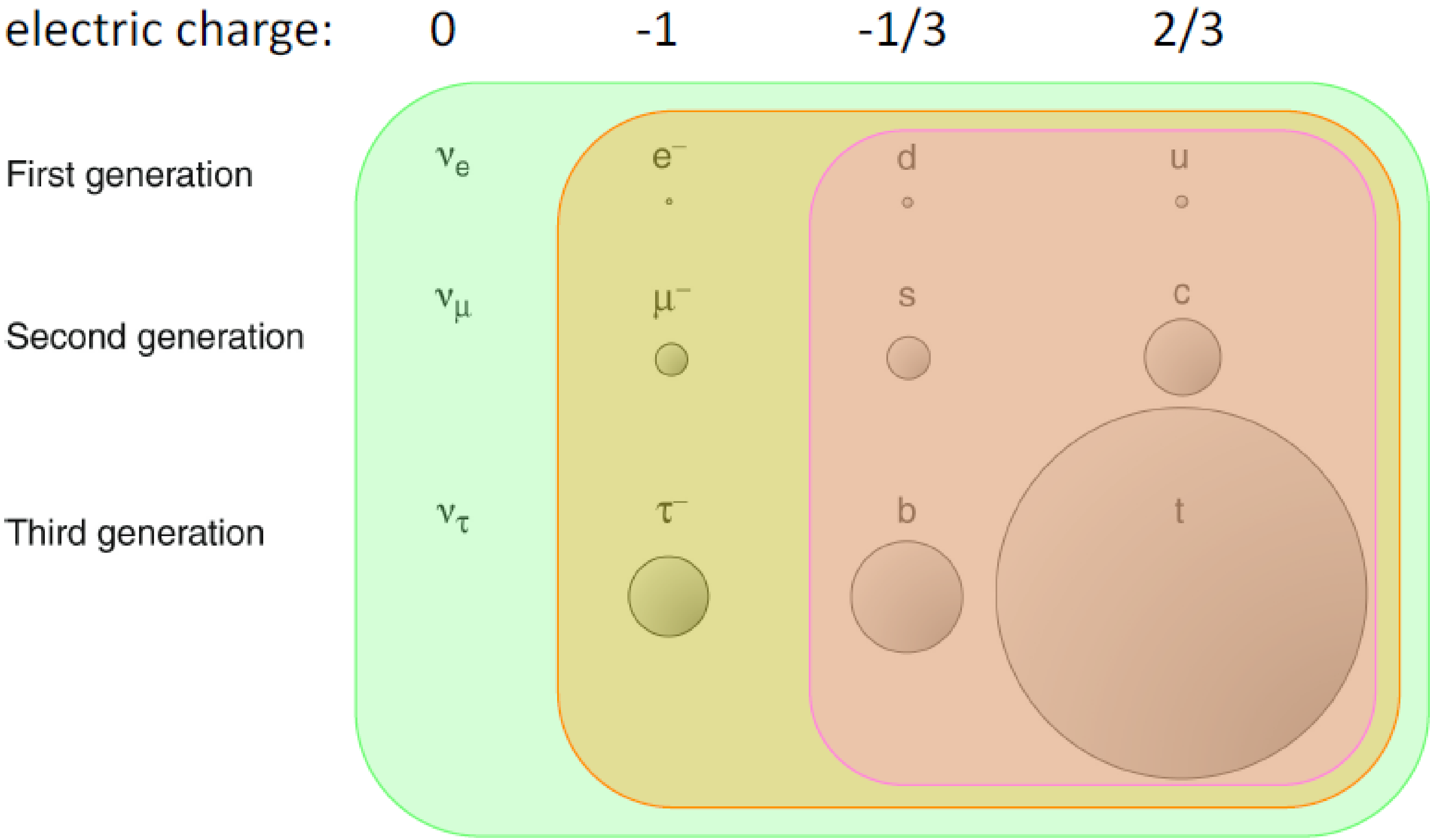


- Rutherford experiment is the first example of the study of the internal structure of an object at the atomic level using a scattering experiment. This approach was developed further with the appearance of particle accelerators
- Up to now we:
  - recapped relativistic kinematics and point body movement in a central force field
  - got acquainted with the concept of an interaction cross section
  - saw how from a scattering experiment it is possible to extract several fundamental properties of matter



Particle decays and nonelementary particles

Reminder: Elementary particles and their interactions



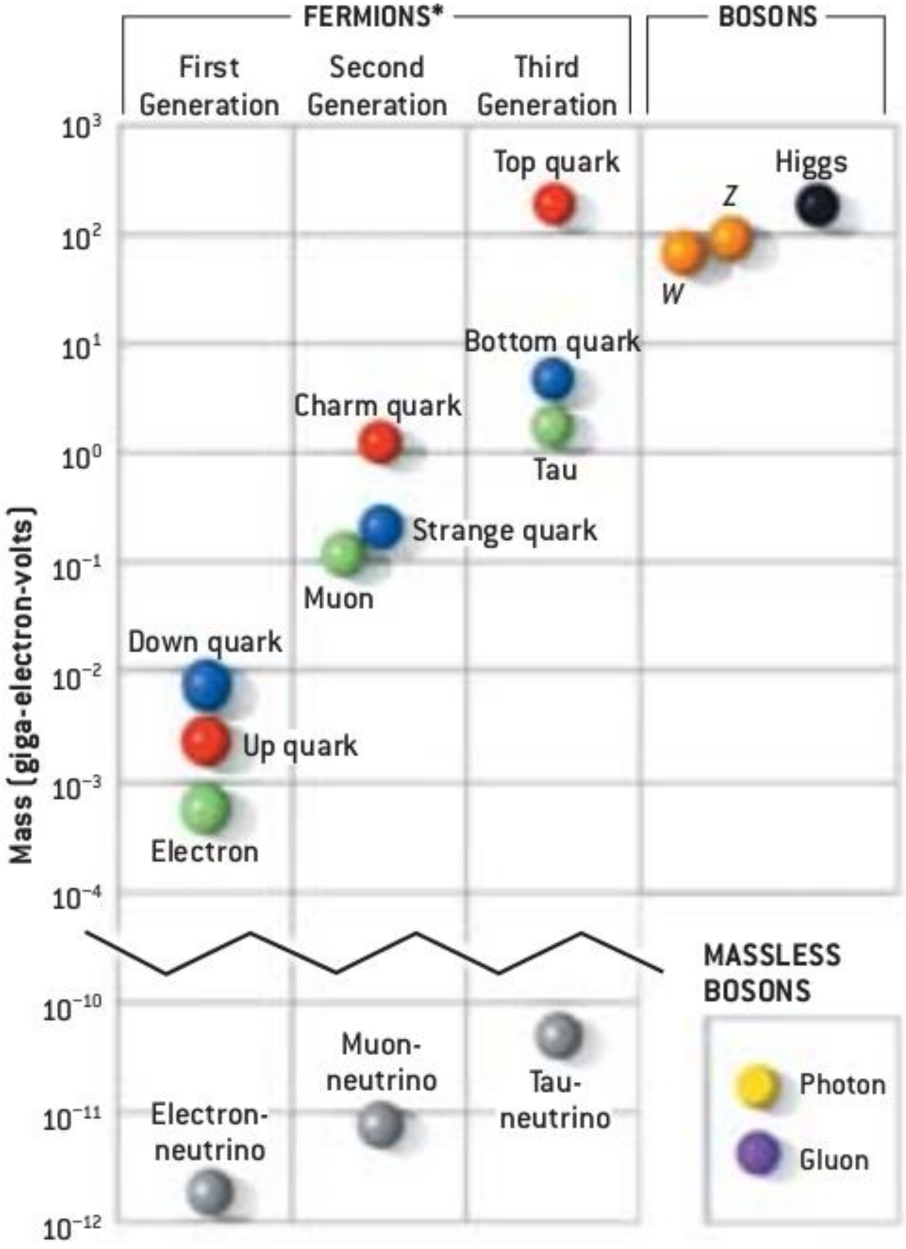
H: **Higgs**

$W^\pm$, Z: **weak**

γ: **electromagnetic**

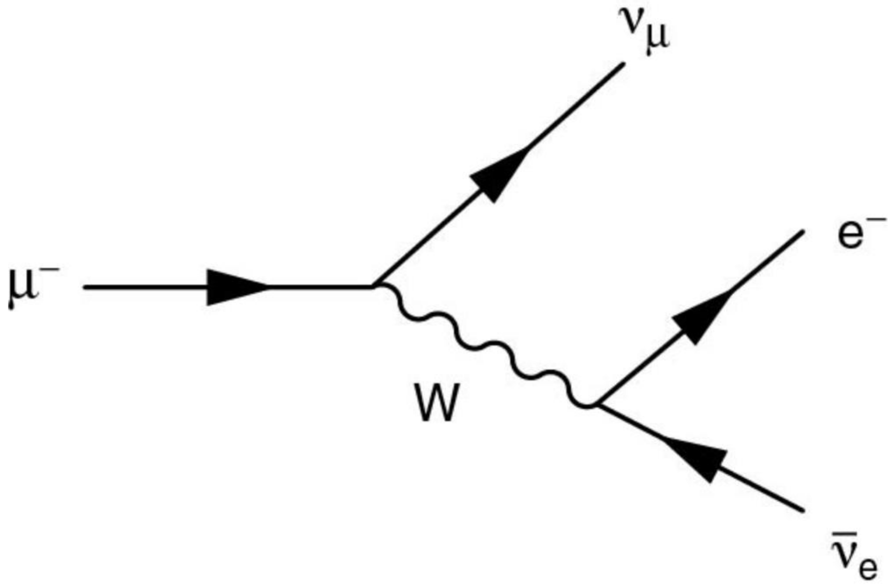
g: **strong**

Reminder: Elementary particle masses



Elementary particle decays

- All decays of *elementary matter particles* go via weak charged current ($W^{\pm}$): only flavour-changing interaction
- For a decay to occur, there need to be particles with lower mass (energy conservation) and it must not be forbidden by symmetries (e.g. p decay forbidden due to baryon number conservation)
- The electron is the lightest charged particle $\Longrightarrow$ nothing to decay to $\Longrightarrow$ the electron is stable
- Example of a muon decay: $\mu^- \rightarrow e^- \, \overline{\nu_e} \nu_\mu$

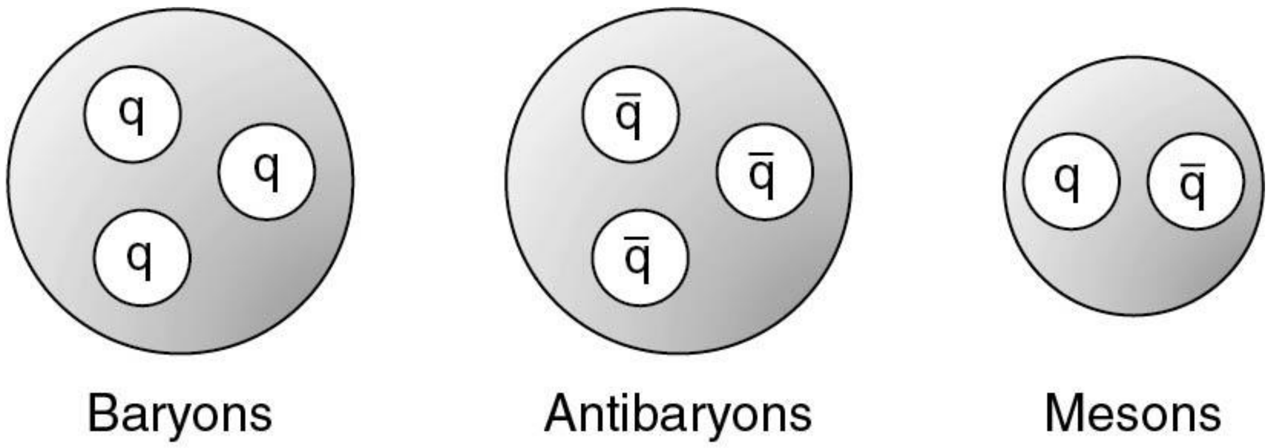


Note an antiparticle $\overline{\nu_e}$ in the decay to conserve lepton number

- The neutrinos are also stable particles

Nonelementary particles

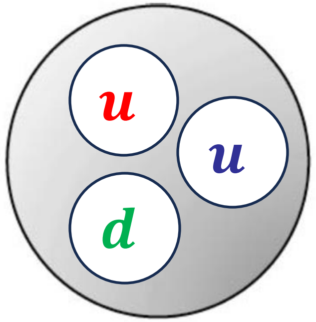
- Quarks form bound states – **hadrons** – thanks to the strong force
  - Baryons – half-integer spin: three quarks ($q_1 q_2 q_3$, e.g. p, n, Δ), pentaquarks (also referred to as exotic baryons)
  - Mesons – integer spin: quark-antiquark pair ($q_1 \overline{q_2}$, e.g. $\pi, \rho, K, \mathrm{D}, \mathrm{B}$), tetraquarks (also referred to as exotic mesons)



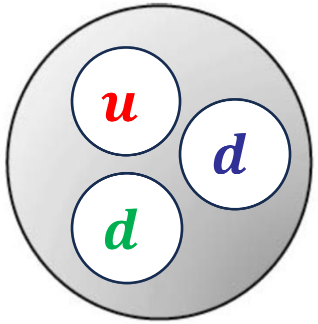
- Quarks are *fermions* ⟹ they cannot have identical quantum numbers in a baryon
  - Should quark flavours always be different in a baryon?
- Baryons made of 3 identical quarks exist ⟹ another quantum number is needed: **colour**
- Quark colour is changed by the strong interaction during a gluon emission

Lightest hadrons

Proton ($q = +1$)



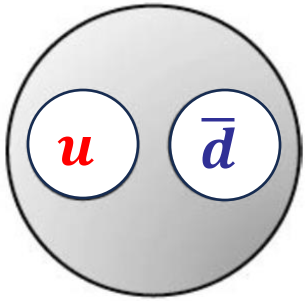
Neutron ($q = 0$)



- Proton $|p> = |uud>$ and neutron $|n> = |udd>$
  - the proton is stable: baryon number conservation in the SM
  - the neutron decays to a proton: $n \to pe^- \bar{\nu}_e$ (the underlying process at quark level is $d \to ue^- \bar{\nu}_e$)
  - all other baryons also (eventually) decay to a proton (plus other particles), not necessarily via weak transition

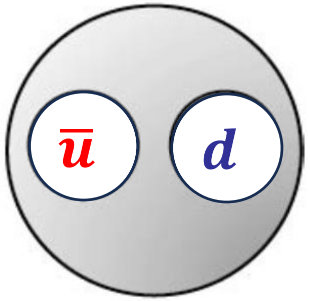
π^+ ($q = +1$)

$|\pi^+> = |u\bar{d}>$



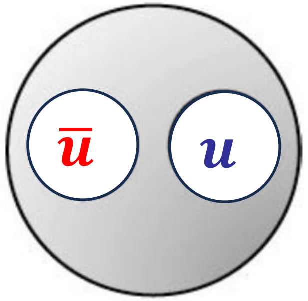
π^- ($q = -1$)

$|\pi^-> = |\bar{u}d>$



π^0 ($q = 0$)

$$|\pi^0> = \frac{1}{\sqrt{2}} |u\bar{u} - d\bar{d}>$$

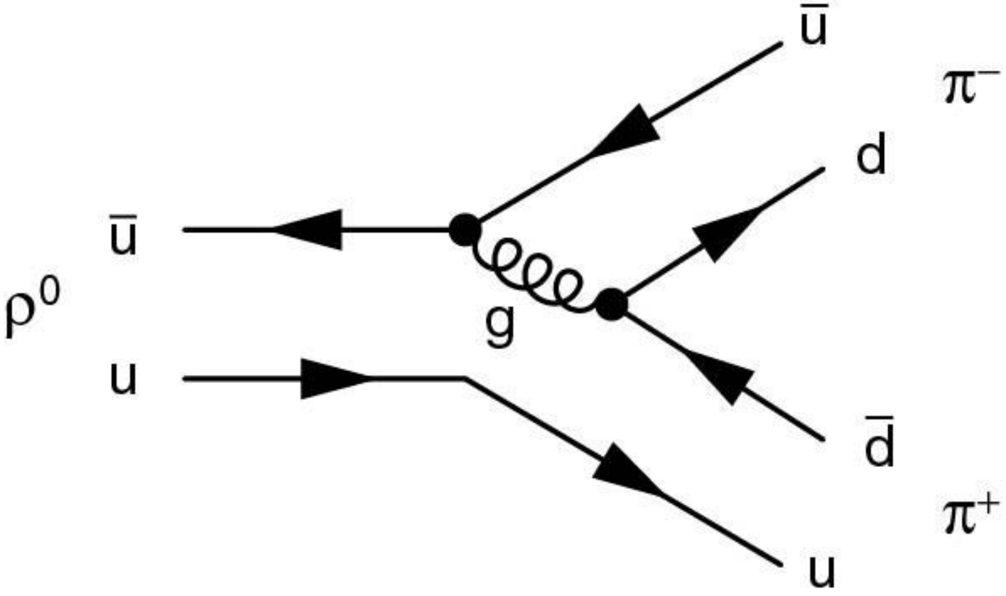


- Pions ($\pi^\pm, \pi^0$)
  - $\pi^+ \to \mu^+ \nu_\mu$ – dominating decay mode (weak process: quark annihilation into W)
  - $\pi^0 \to \gamma\gamma$ – dominating decay mode (electromagnetic process)

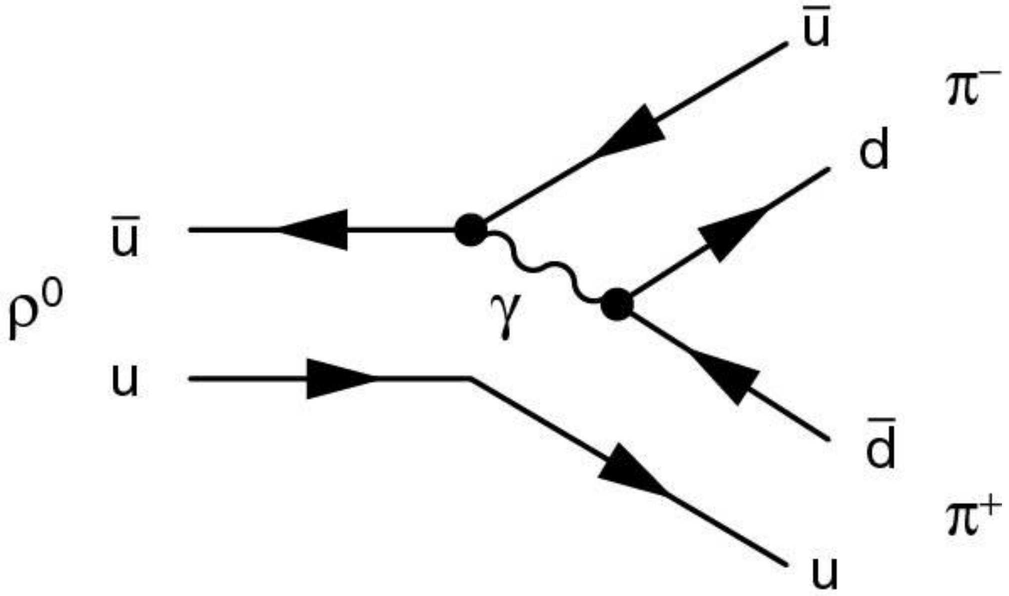
Nonelementary particle decays

- Numerous decay modes are possible (see in the PDG)
- Relative strength of the decay depends on the process involved in the decay

via strong interaction $\propto \alpha_s^2$



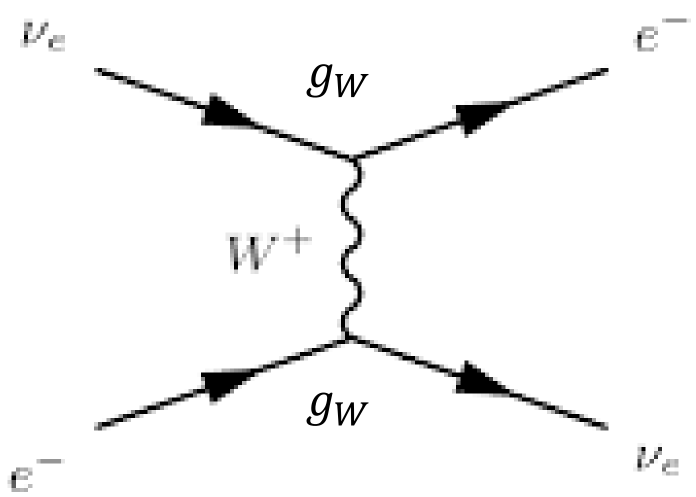
via electromagnetic interaction $\propto \alpha^2$



- As $\alpha_s \sim 1$ and $\alpha \sim 10^{-2}$, decays via strong interaction dominate
- In the same way, decays via electromagnetic interaction dominate over the weak one

Reminder: Known forces

Force	Strength	Bosons		Spin	m/GeV
Strong	1	8 gluons	g	1	0
Electromagnetic	10^{-3}	Photon	γ	1	0
Weak	10^{-8}	W boson Z boson	$W^{\pm}$ Z	1 1	80.4 91.2
Gravitational	10^{-37}	Graviton?	G	2	0

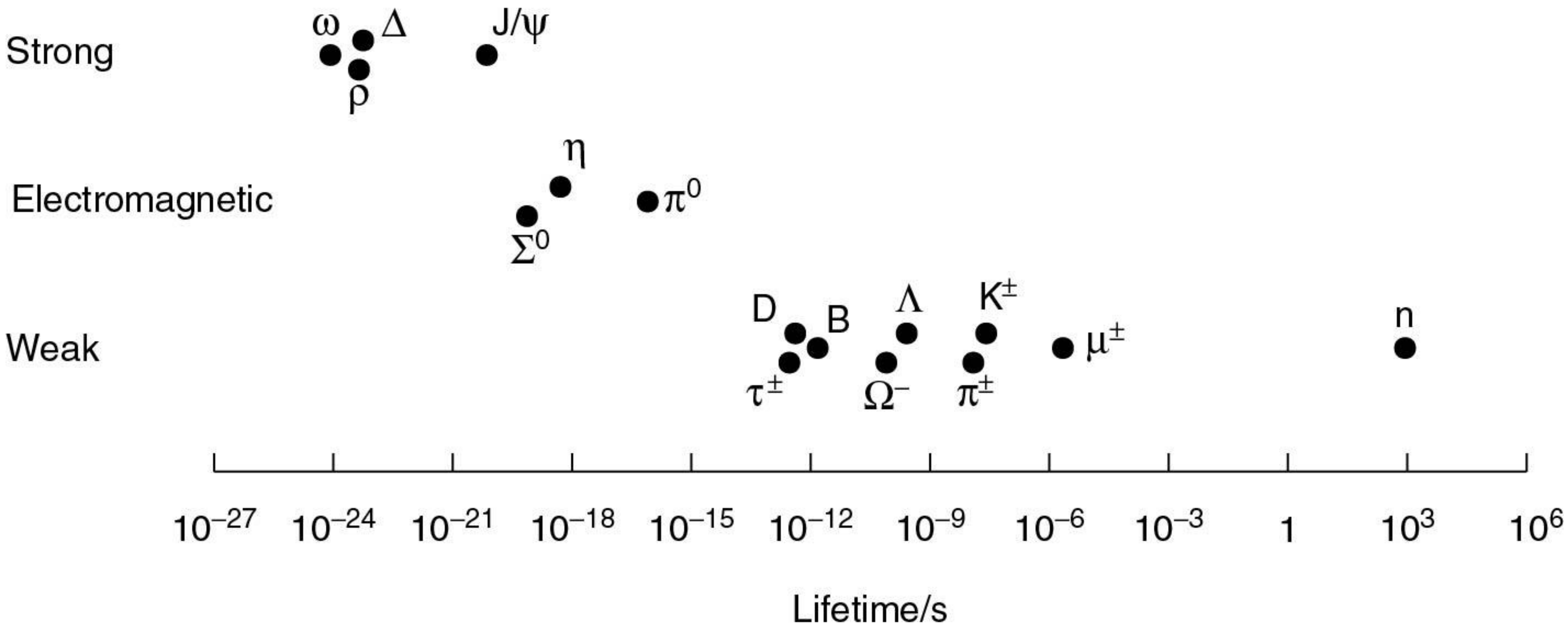


Note: interaction strength only indicative – it depends on the considered distance and energy scales

- Strength of the fundamental interaction represented by the charge g
- Related to the dimensionless coupling "constant" α, e.g. QED
  - $g_{em} = e = \sqrt{4\pi\alpha\epsilon_0\hbar c} = \sqrt{4\pi\alpha}$ (natural units)
- At vertex level: $\alpha_S = 1, \alpha = \frac{1}{137}, \alpha_{W/Z} = 1/30$
- Strength in the table above: effective strength as in particles decays, taking into account the masses of the W/Z bosons in the decay

Particle lifetime

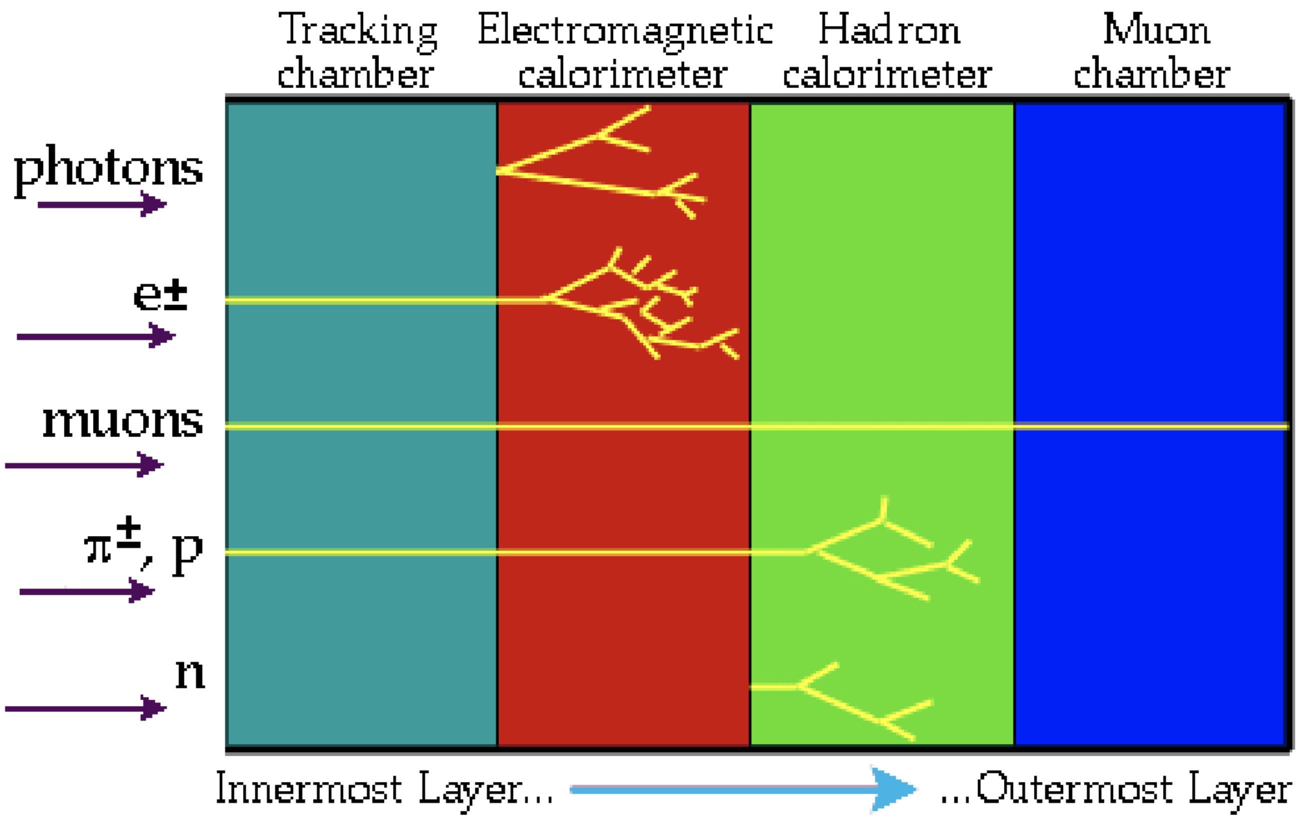
- Order of magnitude of a particle lifetime depends on the processes available for decay:
  - $\pi^{\pm}$ and n decay only via the **weak** interaction $\Longrightarrow$ large lifetimes, **long-lived**
  - π^{0} decays **electromagnetically** $\Longrightarrow$ **intermediate lifetime**
  - ρ can decay via the **strong** interaction $\Longrightarrow$ very short lifetime, **short-lived**



How would you (conceptually) measure the neutron/proton lifetime?

Measurement of particle properties

- To understand the underlying process we have to measure the properties of the particles in the event
  - most heavy particles decay shortly after production
  - particles seen in the detectors: photons, electrons, muons, pions, kaons, protons, neutral hadrons

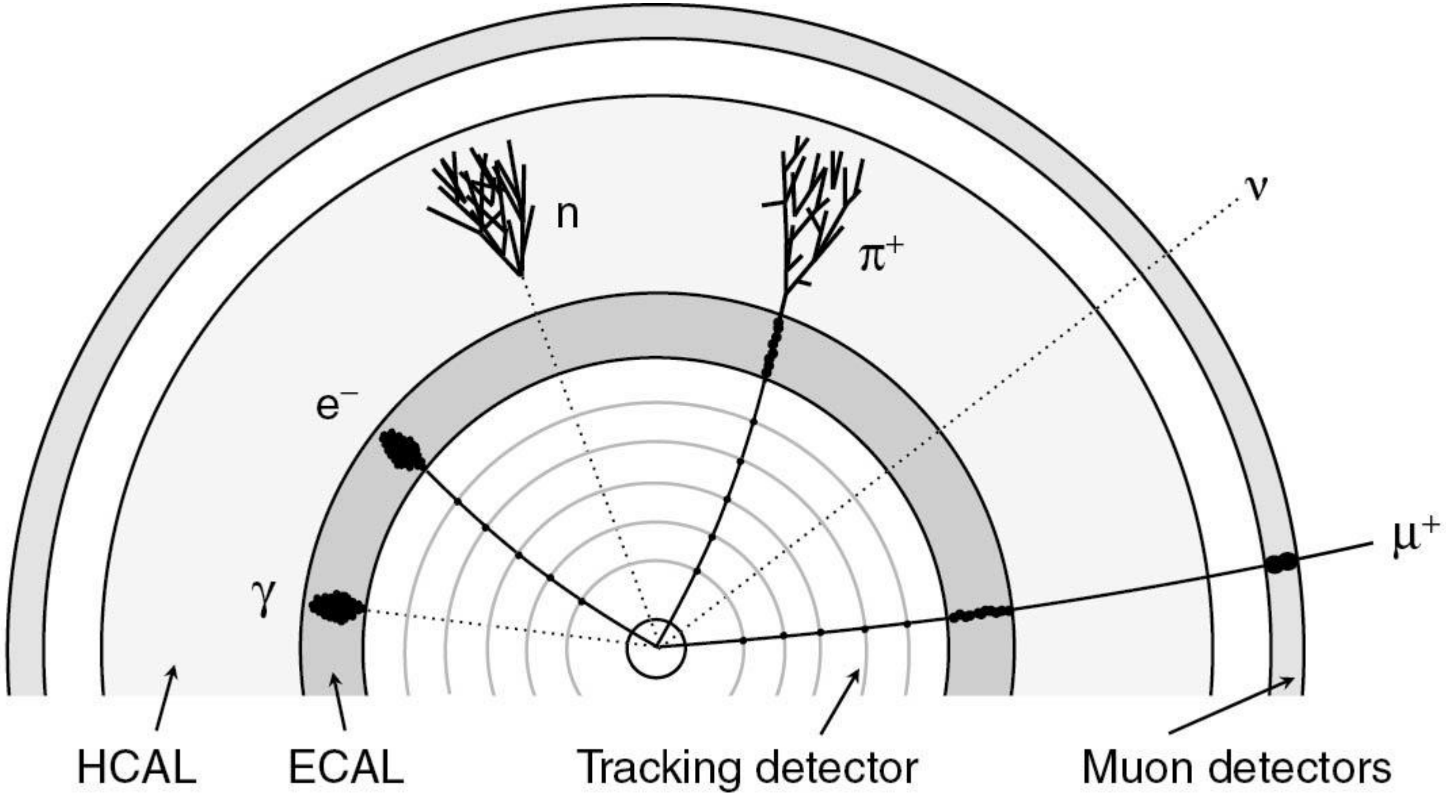


We combine the information about each particles from complex particle detectors to deduce the initial picture.

Measurement of particle properties

- To understand the underlying process we have to measure the properties of the particles in the event
  - most heavy particles decay shortly after production
  - particles seen in the detectors: photons, electrons, muons, pions, kaons, protons, neutral hadrons

center-of-mass experiments



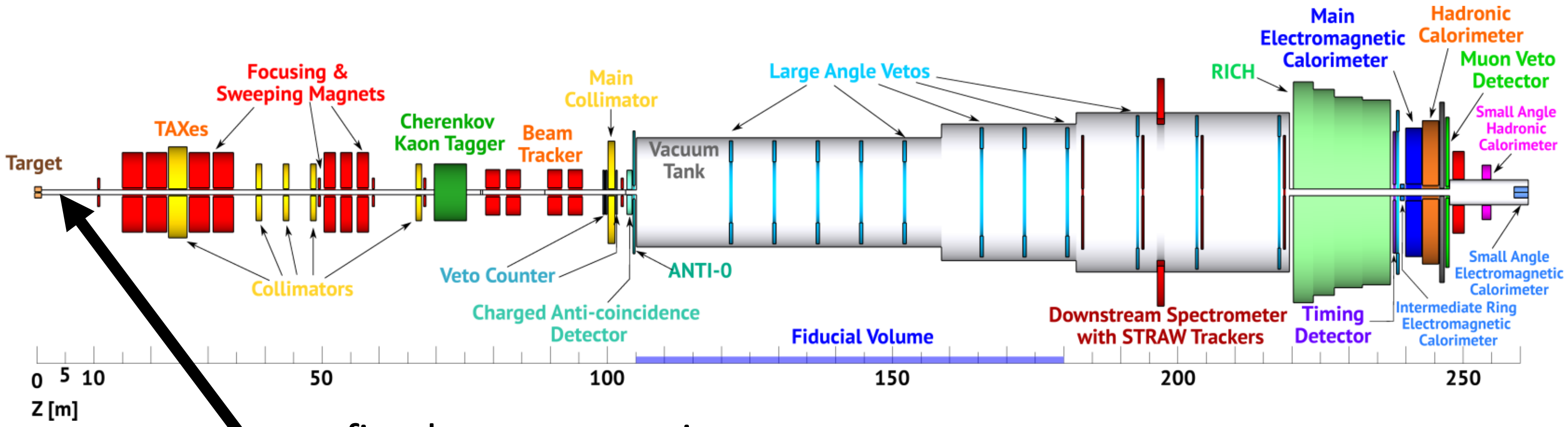
We combine the information about each particles from complex particle detectors to deduce the initial picture.

Measurement of particle properties

- To understand the underlying process we have to measure the properties of the particles in the event
  - detect the decay products and reconstruct the mother particle(s)
- Complete information about each particle in the event is needed
  - charge
  - energy and momentum
  - particle type (so called particle identification or PID)
  - production position of the particle (called production vertex) and decay position (called decay vertex)
  - type of the mother particle
- When we have the properties of all "stable" particles in the detector, we try to reconstruct the initial process in a collision

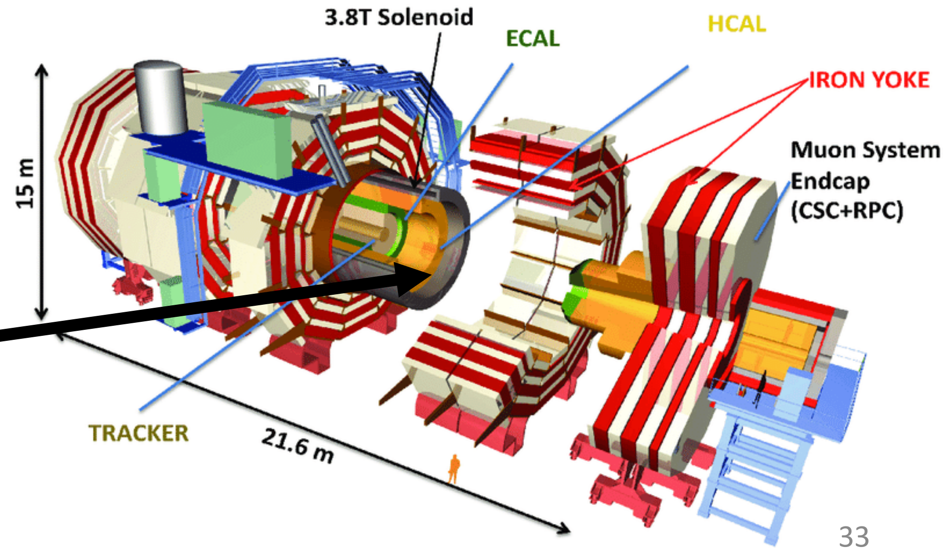
Measurements at particle accelerators

- Two types of high-energy particle accelerator experiments
  - collider experiments: colliding beam machines where two beams of accelerated particles are brought to a collision
  - fixed-target experiments: a single beam is fired at a stationary target



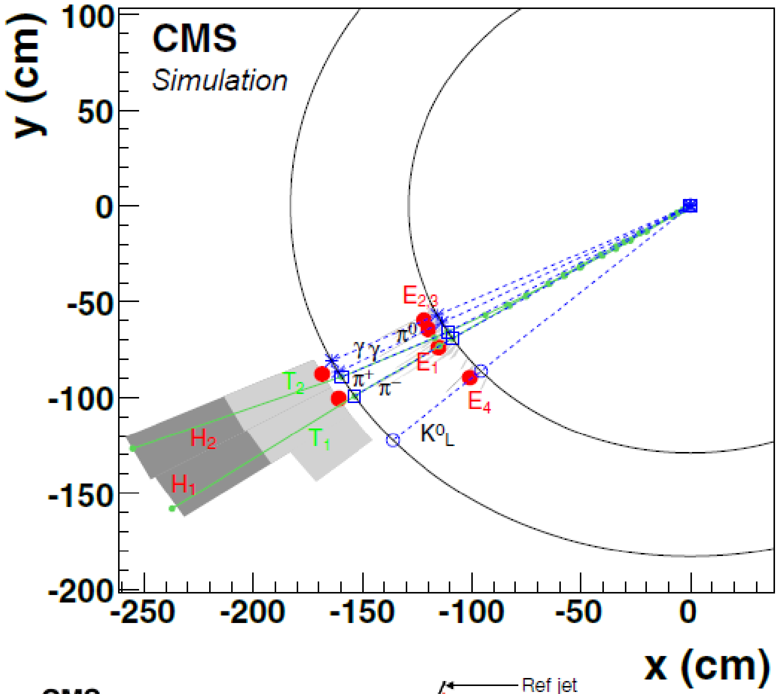
fixed-target experiment
lab frame

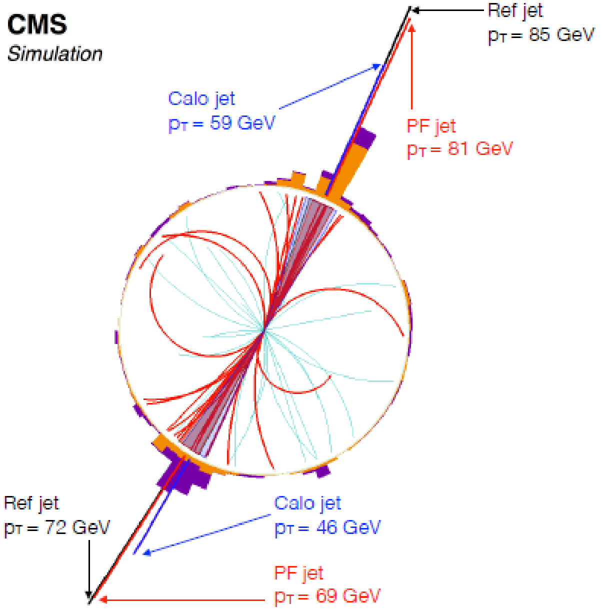
collider experiment
center-of-mass frame



collision point

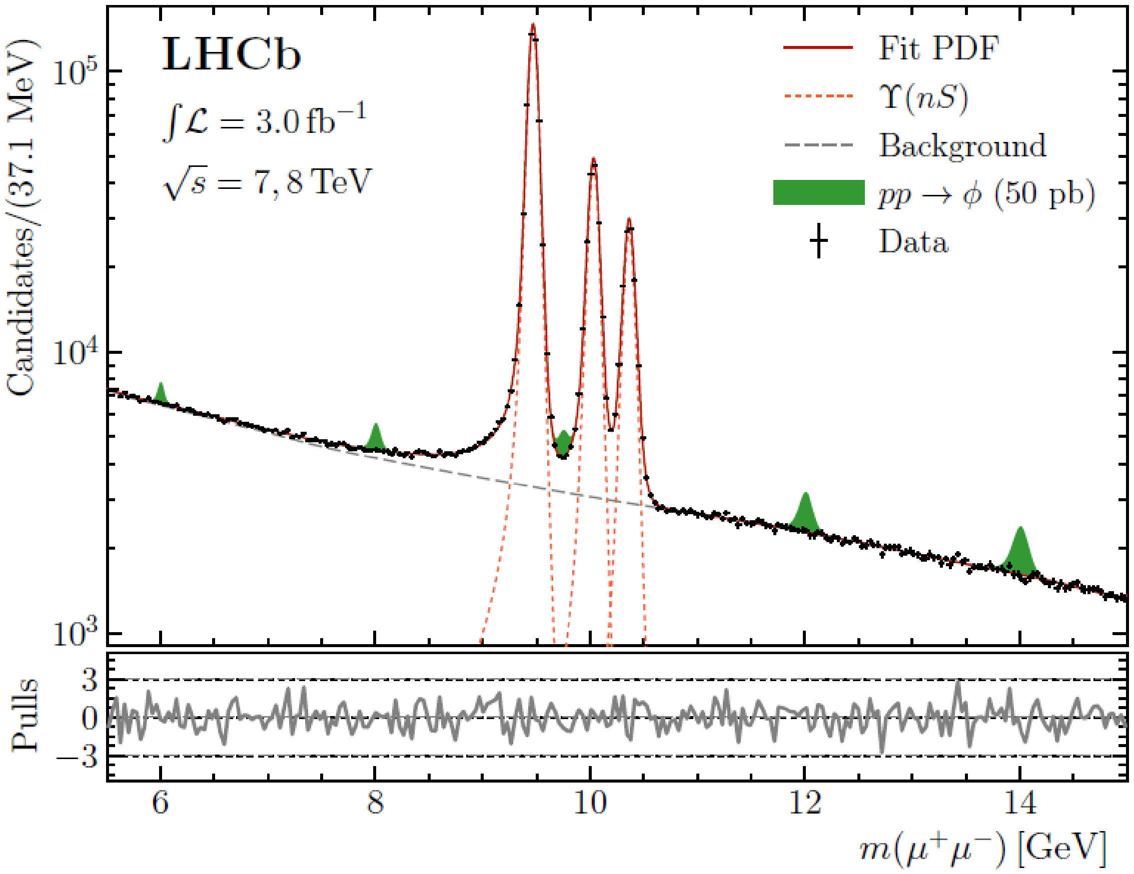
Reconstruction using kinematics





- Deduce the properties of short-lived particles by kinematic reconstruction from the measured momenta and energies of their decay products
- Mass of the decayed particle:
  - $\boldsymbol{M^2 = (\sum E_i)^2 - |\sum \vec{p_i}|^2}$ (sum goes over all daughter particles)
- To measure $\boldsymbol{M^2}$ we need to
  - either measure both energies and momenta of all particles – usually not possible or lacks precision
  - or measure a combination: energy + mass or momentum+mass, where mass is inferred from the particle type $(\pi, K, \mu, e, p, \text{etc.})$

Example of particle properties measurement: dimuons

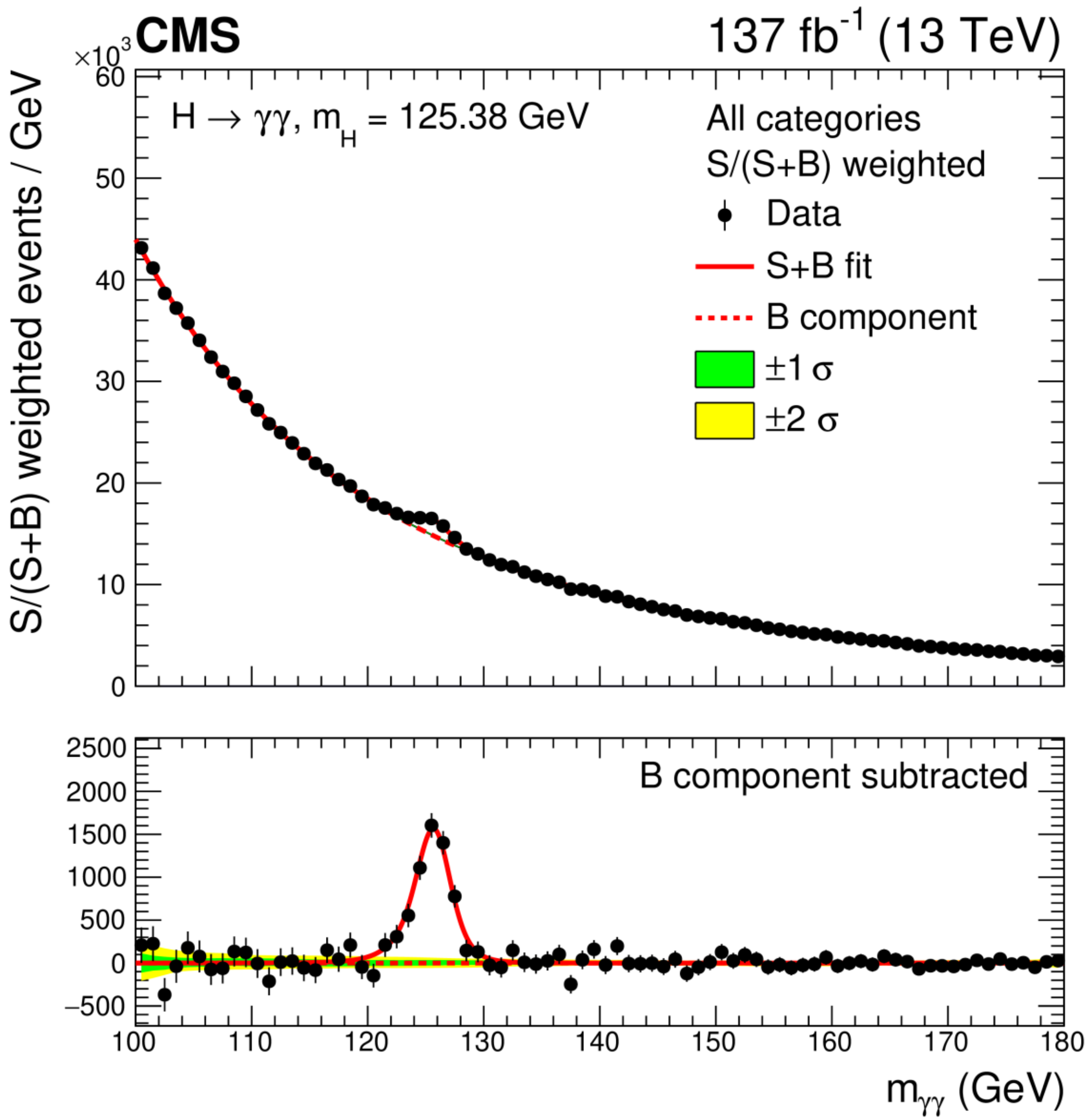


- Decayed particle mass determination:
  - measure the momentum of the μ^+ and μ^-
  - calculate muon energy: $E^2_{\mu^\pm} = m^2_\mu + |\vec{p}_{\mu^\pm}|^2$
  - calculate mass of the decayed particle:

$$\mathbf{M^2 = (E_{\mu^+} + E_{\mu^-})^2 - |\vec{p}_{\mu^+} + \vec{p}_{\mu^-}|^2}$$

- Result
  - Narrow peaks: known Υ resonances
  - Smoothly falling distribution: background from random muon combinations
  - Green peaks: example of signals from new particle (not observed in data)

Example of particle properties measurement: diphotons



- To observe and measure the Higgs boson, look at various decay modes
- $H \rightarrow \gamma\gamma$ is one of the "easiest" channels because it allows to fully reconstruct the mass peak
- Photons must be well-measured!
- Use energy-momentum conservation:
  - $(E, \vec{p})_H = (E_1 + E_2, \vec{p}_1 + \vec{p}_2)$
- Compute $M_H \approx 125\ \mathrm{GeV}/c^2$

Main challenge: how to measure precisely the properties of the daughter particles?

Resonance width

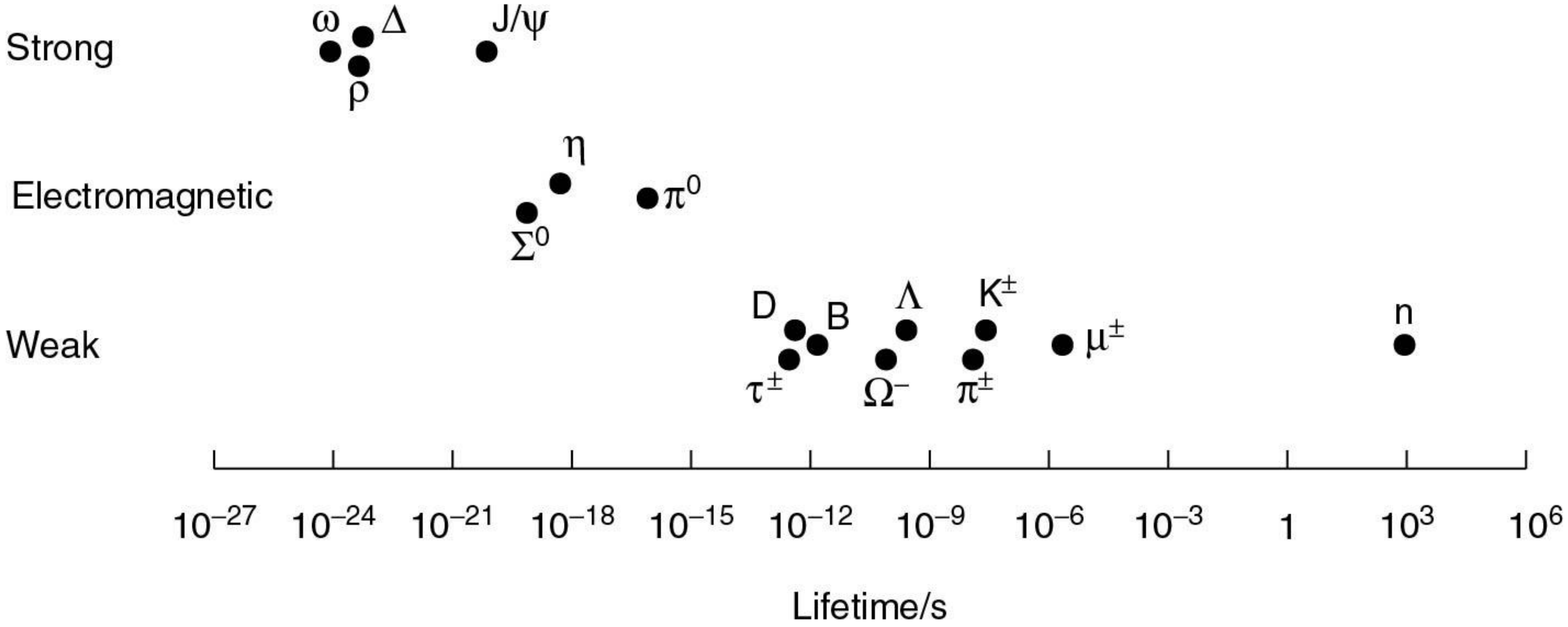
- The observed resonance shape is formed by two effects
- **Particle decay width $\boldsymbol{\Gamma = 1/\tau}$:**
  - resonance shape governed by the relativistic Breit-Wigner distribution:

$$f(E) \propto \frac{M\Gamma^2}{(E^2 - M^2)^2 + M^2\Gamma^2}$$

  - the larger the Γ (smaller the τ), the wider the resonance
- **Detector resolution:**
  - the mass peak is computed using momentum/energy measurements from the detector
  - the Gaussian uncertainties in these measurement translate into Gaussian widening of a mass peak
- The final form of the resonance is a convolution of the two effects with typically one of them dominating

Particle lifetime: detector point of view

- In a particle physics detector, due to Lorentz boost, there are several different particle categories
  - **"stable"** $\Longrightarrow$ traversing the whole detector without decaying
  - **long-lived** $\Longrightarrow$ decay within the detector, producing a secondary vertex
  - **short-lived or prompt** $\Longrightarrow$ decay close to a collision vertex



Particle lifetime: detector point of view

- **Exercise:** particle with a momentum $p = 100\ \mathrm{GeV}/c$, compute average decay length for
  - muons $\mu^{\pm}$
  - charged kaons $K^{\pm}$
  - $B^{\pm}$ mesons
  - tau leptons $\tau^{\pm}$
  - neutral pion π^0
  - J/ψ
- Which particles are **stable**, **long-lived**, and **short-lived** under such conditions. Consider two cases:
  - center-of-mass experiment, surrounding the primary interaction point, "length" $\sim O(10-20\mathrm{m})$
  - fixed target experiment, decay region $\sim O(100\mathrm{m})$ from the primary target, "length" $\sim O(70\mathrm{m})$
- Reminder:
  - decay length $L = \beta\gamma c\tau$, where τ is the particle's lifetime
  - $\beta\gamma = \frac{p}{m}$

Particle lifetime: detector point of view

- **Exercise:** particle with a momentum $p = 100\ \mathrm{GeV}/c$, compute average decay length for
  - **muons $\boldsymbol{\mu^{\pm}}$:** $p = 100\ \mathrm{GeV/c}\ ; m \approx 100\ \mathrm{MeV}/c^2; c = 10^8 m/s; \tau \approx 2 \times 10^{-6} s \Longrightarrow \boldsymbol{L = 600\ \mathrm{km}}$
  - **charged kaons $\boldsymbol{K^{\pm}}$:** $m \approx 500\ \mathrm{MeV}/c^2, \tau \approx 1 \times 10^{-8} s$
  - **$\boldsymbol{B^{\pm}}$ mesons:** $m \approx 5300\ \mathrm{MeV}/c^2, \tau \approx 1.6 \times 10^{-12} s$
  - **tau leptons $\boldsymbol{\tau^{\pm}}$:** $m \approx 1800\ \mathrm{MeV}/c^2, \tau \approx 2.9 \times 10^{-13} s$
  - **neutral pion $\boldsymbol{\pi^{0}}$:** $m \approx 135\ \mathrm{MeV}/c^2, \tau \approx 1 \times 10^{-18} s$
  - **$\boldsymbol{J/\psi}$:** $m \approx 3000\ \mathrm{MeV}/c^2, \tau \approx 7 \times 10^{-21}\ s$
- Which particles are **stable**, **long-lived**, and **short-lived** under such conditions. Consider two cases:
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Particle lifetime: detector point of view

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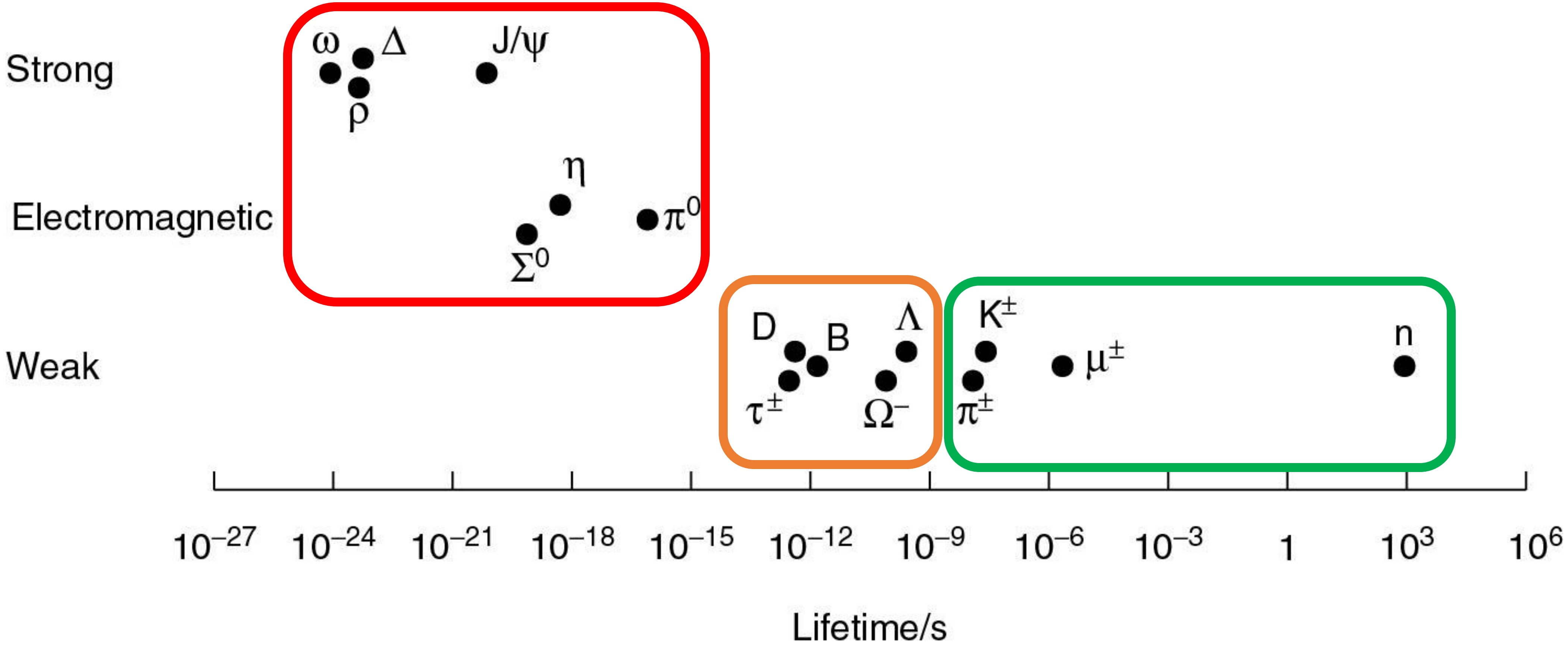

Summary of Lecture II

<u>Main learning outcomes</u>

- Using the Rutherford scattering as an example of how we use scattering experiments to learn more about the nature of the fundamental particles
- Examples of particles decays via the weak, strong and electromagnetic interactions
- Experimental techniques to obtain physics observables from quantities measured at experiments
- Classification of nonelementary particles and their experimental signatures